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Tugas 2 Metmat (c)

Halaman 24

1). g) $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 < x \le \pi \end{cases}$, dengan $f(x+2\pi) = f(x)$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x^2\,dx\right]$$
$$= \frac{1}{\pi}\left[\left[\frac{1}{3}x^3\right]_0^{\pi}\right]$$
$$= \frac{\pi^3}{3\pi}$$
$$= \frac{1}{3}\pi^2$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x^2 \cdot \cos\frac{n\pi x}{\pi}\,dx\right]$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} x^2 \cos nx\,dx\right]$$
$$= \frac{1}{\pi}\left[\left[x^2\left(\frac{1}{n}\sin nx\right) - 2x\left(-\frac{1}{n^2}\cos nx\right) + 2\left(-\frac{1}{n^3}\sin nx\right)\right]_0^{\pi}\right]$$
$$= \frac{1}{\pi}\left[\left(0 + \frac{2\pi}{n^2}(-1)^n - 0\right) - (0 - 0 - 0)\right]$$
$$= \frac{1}{\pi}\cdot\frac{2\pi}{n^2}(-1)^n = \frac{2}{n^2}(-1)^n$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} x^2 \sin\frac{n\pi x}{\pi}\,dx\right]$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} x^2 \sin nx\,dx\right]$$
$$= \frac{1}{\pi}\left[\left[x^2\left(-\frac{1}{n}\cos nx\right) - 2x\left(-\frac{1}{n^2}\sin nx\right) + 2\left(\frac{1}{n^3}\cos nx\right)\right]_0^{\pi}\right]$$
$$= \frac{1}{\pi}\left[\left(\pi^2\left(-\frac{1}{n}\cdot(-1)^n\right) - 2\pi\left(-\frac{1}{n^2}(0)\right) + \frac{2}{n^3}(-1)^n\right) - \left(0 - 0 + \frac{2}{n^3}\right)\right]$$
$$= \frac{1}{\pi}\left[\frac{\pi^2}{n}(-1)^{n+1} + 0 + \frac{2}{n^3}(-1)^n - \frac{2}{n^3}\right]$$
$$= \frac{1}{\pi}\left[\frac{\pi^2}{n}(-1)^{n+1} + \frac{2}{n^3}\left((-1)^n - 1\right)\right]$$

Deret Fourier-nya adalah :

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{3}\pi^2 + \sum_{n=1}^{\infty}\left(\frac{2}{n^2}(-1)^n \cdot \cos nx + \frac{1}{\pi}\left(\frac{\pi^2}{n}(-1)^{n+1} + \frac{2}{n^3}\left((-1)^n - 1\right)\right)\sin nx\right)$$

l). $f(x) = \begin{cases} 2-x, & 0 \le x \le 4 \\ x-6, & 4 < x \le 8 \end{cases}$, dengan $f(x+8) = f(x)$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{4}\left[\int_0^4 (2-x)\,dx + \int_4^8 (x-6)\,dx\right]$$

$$= \frac{1}{4}\left[\int_0^4 2\,dx - \int_0^4 x\,dx + \int_4^8 x\,dx - \int_4^8 6\,dx\right]$$

$$= \frac{1}{4}\left[2x\Big]_0^4 - \frac{1}{2}x^2\Big]_0^4 + \frac{1}{2}x^2\Big]_4^8 - 6x\Big]_4^8\right]$$

$$= \frac{1}{4}\left[8 - 8 + (32-8) - 6(8-4)\right]$$

$$= \frac{1}{4}\left[24 - 24\right]$$

$$= 0$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{4}\left[\int_0^4 (2-x)\cdot\cos\frac{n\pi x}{4}\,dx + \int_4^8 (x-6)\cos\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[2\int_0^4 \cos\frac{n\pi x}{4}\,dx - \int_0^4 x\cos\frac{n\pi x}{4}\,dx + \int_4^8 x\cos\frac{n\pi x}{4}\,dx - 6\int_4^8 \cos\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[2\cdot\frac{4}{n\pi}\sin\frac{n\pi x}{4}\Big]_0^4 - \left(x\cdot\frac{4}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{(n\pi)^2}\cos\frac{n\pi x}{4}\right)\Big]_0^4 + \left(x\cdot\frac{4}{n\pi}\sin\frac{n\pi x}{4} + \frac{16}{(n\pi)^2}\cos\frac{n\pi x}{4}\right)\Big]_4^8\right.$$
$$\left. - 6\cdot\frac{4}{n\pi}\sin\frac{n\pi x}{4}\Big]_4^8\right]$$

$$= \frac{1}{4}\left[\frac{8}{n\pi}(0) - \left(\frac{16}{n\pi}(0) + \frac{16}{(n\pi)^2}\right) + \left(\left(\frac{32}{n\pi}\sin 2n\pi + \frac{16}{(n\pi)^2}\cos 2n\pi\right) - \left(\frac{16}{n\pi}(\sin n\pi) + \frac{16}{(n\pi)^2}\cos n\pi\right)\right)\right.$$
$$\left. - \left(\frac{24}{n\pi}\left(\sin 2n\pi - \sin n\pi\right)\right)\right]$$

$$= \frac{1}{4}\left[0 - \left(\frac{16}{(n\pi)^2}\right) + \left(0 + \frac{16}{(n\pi)^2} - \left(0 + \frac{16}{(n\pi)^2}(-1)^n\right)\right) - (0)\right]$$

$$= \frac{1}{4}\left[-\frac{16}{(n\pi)^2} + \frac{16}{(n\pi)^2} + \frac{16}{(n\pi)^2}(-1)^{n+1}\right]$$

$$= \frac{1}{4}\left[\frac{16}{(n\pi)^2}(-1)^{n+1}\right]$$

$$a_n = \frac{4}{(n\pi)^2}(-1)^{n+1} \begin{cases} \frac{4}{(n\pi)^2}, & n = 1,3,5,\dots \\ -\frac{4}{(n\pi)^2}, & n = 2,4,6,\dots \end{cases}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{4}\left[\int_0^4 (2-x)\sin\frac{n\pi x}{4}\,dx + \int_4^8 (x-6)\sin\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[2\int_0^4 \sin\frac{n\pi x}{4}dx - \int_0^4 x\sin\frac{n\pi x}{4}dx + \int_4^8 x\sin\frac{n\pi x}{4}dx - 6\int_4^8 \sin\frac{n\pi x}{4}dx\right]$$

$$= \frac{1}{4}\left[2.\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)\Big]_0^4 - \left(x.\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right) + \frac{16}{(n\pi)^2}\sin\frac{n\pi x}{4}\Big]_0^4\right)\right.$$

$$\left. + \left(x\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right) + \frac{16}{(n\pi)^2}\sin\frac{n\pi x}{4}\Big]_4^8\right) - 6.\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)\Big]_4^8\right]$$

$$= \frac{1}{4}\left[-\frac{8}{n\pi}\left((-1)^n - 1\right) - \left(\frac{16}{n\pi}(-\cos n\pi) + \frac{16}{(n\pi)^2}\cdot\sin n\pi - \left(0 + \frac{16}{(n\pi)^2}\sin 0^\circ\right)\right)\right.$$

$$+ \left(\left(\frac{32}{n\pi}(-\cos 2n\pi) + \frac{16}{(n\pi)^2}\sin 2n\pi\right) - \left(\frac{16}{n\pi}(-\cos n\pi) + \frac{16}{(n\pi)^2}\sin n\pi\right)\right)$$

$$\left. + \left(\frac{24}{n\pi}\left(\cos 2n\pi - \cos n\pi\right)\right)\right]$$

$$= \frac{1}{4}\,\frac{8}{n\pi}\left[\left(1 + (-1)^{n+1}\right) + \left(2(-1)^n + 0 - 0\right) + \left(\left(4(-1) + 2(0)\right) + \left(2(-1)^n + 0\right)\right)\right.$$

$$\left. + \left(3\left(1 + (-1)^{n+1}\right)\right)\right]$$

$$= \frac{2}{n\pi}\left[1 - 4 + 3 + (-1)^{n+1} + 2(-1)^n + 2(-1)^n + 3(-1)^{n+1}\right]$$

$$= \frac{2}{n\pi}\left[0 + 4(-1)^{n+1} + 4(-1)^n\right]$$

$$= \frac{8}{n\pi}\left[(-1)^{n+1} + (-1)^n\right]$$

$$= \frac{8}{n\pi}\left[-(-1)^n + (-1)^n\right]$$

$$= 0$$

▶ Deret Fourier-nya adalah

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$= \frac{1}{2}(0) + \sum_{n=1}^{\infty}\left(\frac{4}{(n\pi)^2}(-1)^{n+1}\, s\frac{n\pi x}{4} + 0\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{4}{(n\pi)^2}(-1)^{n+1}\cos\frac{n\pi x}{4}\right)$$

$$= \frac{4}{\pi^2}\cos\frac{\pi x}{4} - \frac{1}{\pi^2}\cos\frac{\pi x}{2} + \frac{4}{9\pi^2}\cos\frac{3\pi x}{4} - \frac{4}{16\pi^2}\cos\pi x + \dots$$

Soal Tambahan :

$$f(x) = \begin{cases} \cos x, & 0 \le x < \pi \\ 0, & \pi \le x \le 2\pi \end{cases}, \quad f(x) = f(x+2\pi)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} \cos x\,dx + \int_{\pi}^{2\pi} 0\,dx\right]$$

$$= \frac{1}{\pi}\left[\sin x\Big]_0^{\pi}\right]$$

$$= \frac{1}{1}(0)$$

$$= 0$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

Catatan :

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b \quad +$$
$$\sin(a+b) + \sin(a-b) = 2\sin a\cos b$$

- $\sin a\cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$
- $\cos a\sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b \quad +$$
$$\cos(a+b) + \cos(a-b) = 2\cos a\cos b$$

- $\cos a\cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$
- $\sin a\sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\int_0^{\pi} \cos x \cdot \cos nx\,dx + \int_{\pi}^{2\pi} 0\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{2}\int_0^{\pi} \cos(1+n)x + \cos(1-n)x\,dx\right]$$

$$= \frac{1}{2\pi}\left[\left(\int_0^{\pi}\cos(1+n)x\,dx\right) + \left(\int_0^{\pi}\cos(1-n)x\,dx\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{1+n}\sin(1+n)x\Big]_0^{\pi} + \frac{(-1)}{1-n}\sin(1-n)x\Big]_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[0+0\right]$$

$$= 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x).\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi}\cos x\,\sin\frac{n\pi x}{\pi}\,dx + \int_{\pi}^{2\pi} 0\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{2}\int_0^{\pi}\sin(1+n)x - \sin(1-n)x\,dx\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\sin(1+n)x\,dx - \int_0^{\pi}\sin(1-n)x\,dx\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{1+n}\left(-\cos(1+n)x\right)\Big]_0^{\pi} - \frac{1}{(1-n)}\left(\cos(1-n)x\right)\Big]_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{1}{1+n}\left((-1)^{n+1}-1\right) - \frac{1}{1-n}\left((-1)^{1-n}-1\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{1+n}\left((-1)^n+1\right) + \frac{1}{1-n}\left(1+(-1)^n\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{(1-n)(1+(-1)^n) + (1+(-1)^n)(1+n)}{1-n^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1+(-1)^n \cancel{-n} \cancel{-n(-1)^n} + 1 + (-1)^n \cancel{+n} \cancel{+n(-1)^n}}{1-n^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2(1+(-1)^n)}{1-n^2}\right]$$

$$= \frac{1}{\pi}\left(\frac{1+(-1)^n}{1-n^2}\right) = \begin{cases} 0, & n = 1, 3, 5, \ldots \\ \dfrac{2}{\pi(1-n^2)}, & n = 2, 4, 6, \ldots \end{cases}$$

Deret Fourier :

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$= 0 + \sum_{n=1}^{\infty}\left(0 + \frac{1+(-1)^n}{(1-n^2)\pi}\right.$$

$$= \sum_{n=1}^{\infty}\frac{1+(-1)^n}{\pi(1-n^2)}\sin nx$$

$$f(x) = -\frac{2}{\pi(-3)}.\sin 2x + 0 - \frac{2}{\pi(-15)}\sin 4x + 0 - \frac{2}{\pi(-35)}\sin 6x + \ldots$$

$$= \frac{2}{3\pi}\sin 2x + \frac{2}{15\pi}\sin 4x + \frac{2}{35\pi}\sin 6x + \ldots$$

2). Hasil dari penderetan menurut Fourier soal no. 1a untuk $0 < x < \pi$ adalah

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$$

Untuk nilai x berapakah, sehingga dapat ditunjukkan nilai deret berikut ini adalah benar

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$$

$$f(x) - \frac{1}{2} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$$

$$\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} = \frac{\pi}{2}\left(f(x) - \frac{1}{2}\right)$$

\# Deret pada soal : $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

maka,

$$\frac{\cancel{\pi}}{\cancel{2}}\left(f(x) - \frac{1}{2}\right) = \frac{\cancel{\pi}}{\cancel{4}\,2}$$

$$f(x) = 1$$

$$\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1} = \frac{\sin x}{1} + \frac{\sin 3x}{3} + \dots$$

Agar menjadi deret $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$, maka dibutuhkan nilai x yang menghasilkan nilai 1. Diperoleh $x = \frac{\pi}{2}$

Jawab : $x = \frac{\pi}{2}$ //

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Tugas DF Metmat (0)

6). Diberikan fungsi $f(x) = e^{-x}$

a). Dapatkan deret Fourier dari $f(x)$ untuk $-\pi \le x \le \pi$

$$\Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}dx$$

$$= \frac{2}{\pi}\int_0^{\pi} e^{-x}dx$$

$$= \frac{2}{\pi}\left[\left[-e^{-x}\right]_0^{\pi}\right]$$

$$= \frac{2}{\pi}\left[-e^{-\pi} + 1\right]$$

$$= \frac{2}{\pi}\left[1 - \frac{1}{e^{\pi}}\right] //$$

$$\Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-x}\cdot\cos\frac{n\pi x}{\pi}dx$$

$$= \frac{2}{\pi}\int_0^{\pi} e^{-x}\cdot\cos(nx)\,dx$$

$*$ misal : $\int_0^{\pi} e^{-x}\cos(nx)\,dx = T$

misal : $u = \cos(nx)$ $dv = e^{-x}dx$

$du = n(-\sin(nx))dx$ $v = -e^{-x}$

$du = -n\sin(nx)dx$

$$\Rightarrow T = u.v - \int_0^{\pi} v\,du$$

$$= -e^{-x}.\cos(nx)\Big]_0^{\pi} - \int_0^{\pi}(-e^{-x}).(-n\sin(nx))dx$$

$$= \left(-\frac{1}{e^{\pi}}\cdot\cos n\pi + 1\right) - n\int_0^{\pi} e^{-x}\sin(nx)dx$$

$$\Rightarrow \int_0^{\pi}\underbrace{\sin(nx)}_{u}\ \underbrace{e^{-x}dx}_{dv}$$

misal : $u = \sin(nx)$ $dv = e^{-x}dx$

$du = n\cos(nx)\,dx$ $v = -e^{-x}$

$$\Rightarrow \int_0^{\pi} \sin(nx)\, e^{-x} dx = -e^{-x} \cdot \sin(nx) \Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}) \cdot n \cos(nx)\, dx$$

$$= \left(-e^{-\pi} \cdot (0) - 0\right) + n \int_0^{\pi} e^{-x} \cdot \cos(nx)\, dx$$

$$= n\,T$$

$$T = \left(-\frac{1}{e^{\pi}} \cos(n\pi) + 1\right) - n\,(nT)$$

$$T = \left(\frac{e^{\pi} - \cos(n\pi)}{e^{\pi}}\right) - n^2 T$$

$$T\,(n^2+1) = \left(\frac{e^{\pi} - \cos(n\pi)}{e^{\pi}}\right)$$

$$T = \left(\frac{e^{\pi} - \cos(n\pi)}{(n^2+1)\, e^{\pi}}\right)$$

$$a_n = \frac{2}{\pi} \cdot T$$

$$= \frac{2}{\pi} \cdot \left[\frac{e^{\pi} - \cos(n\pi)}{(n^2+1)\, e^{\pi}}\right]$$

$$a_n = \begin{cases} \frac{2}{\pi}\left[\frac{e^{\pi}+1}{(n^2+1)e^{\pi}}\right], & n \text{ ganjil} \\ \frac{2}{\pi}\left[\frac{e^{\pi}-1}{(n^2+1)e^{\pi}}\right], & n \text{ genap} \end{cases}$$

$$\cdot\!> \quad b_n = \frac{1}{\pi} \int_{-L}^{L} e^{-x} \cdot \sin(nx)\, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} e^{-x} \sin(nx)\, dx$$

$\divideontimes$ Anggap $\int_0^{\pi} e^{-x} \sin(nx)\, dx = P$

misal : $u = \sin(nx)$ $\qquad dv = e^{-x} dx$

$du = n \cos(nx) dx \qquad v = -e^{-x}$

$$P = -e^{-x} \cdot \sin(nx) \Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}) \cdot n \cos(nx) dx$$

$$= 0 + n \int_0^{\pi} e^{-x} \cdot \cos nx\, dx$$

$\divideontimes\divideontimes$ $\int_0^{\pi} e^{-x} \cos nx\, dx$

misal : $u = \cos(nx)$ $\qquad dv = e^{-x} dx$

$du = -n \sin(nx) dx \qquad v = -e^{-x}$

$$\Rightarrow (-e^{-x}) \cdot \cos(nx) \Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}) (-n \sin(nx)) dx$$

$$= \left[\left(-\frac{1}{e^{\pi}} \cos n\pi\right) + 1\right] - nP$$

Jadi, $P = n\left[\left(-\frac{\cos n\pi}{e^\pi}\right) + 1 - nP\right]$

$$P = n - \frac{n\cos n\pi}{e^\pi} - n^2 P$$

$$P(n^2+1) = \frac{ne^\pi - n\cos n\pi}{e^\pi}$$

$$P = \frac{n(e^\pi - \cos n\pi)}{(n^2+1)e^\pi}$$

$$b_n = \frac{2}{\pi} \cdot P$$

$$= \frac{2n(e^\pi - \cos n\pi)}{\pi e^\pi (n^2+1)}$$

$$b_n = \begin{cases} \dfrac{2n(e^\pi + 1)}{\pi e^\pi (n^2+1)} & , n \text{ ganjil} \\ \dfrac{2n(e^\pi - 1)}{\pi e^\pi (n^2+1)} & , n = \text{genap} \end{cases}$$

Jawab : $f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}\right)$

$$= \frac{1}{2} \cdot \frac{2}{\pi}\left[1 - \frac{1}{e^\pi}\right] + \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\left[\frac{e^\pi - (-1)^n}{(n^2+1)e^\pi}\right]\cos(nx) + \frac{2n(e^\pi - (-1)^n)}{\pi e^\pi (n^2+1)}\sin(nx)\right)$$

$$= \frac{1}{\pi}\left[\frac{e^\pi - 1}{e^\pi}\right] + \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\left[\frac{e^\pi - (-1)^n}{(n^2+1)e^\pi}\right]\cos(nx) + \frac{2n(e^\pi - (-1)^n)}{\pi e^\pi (n^2+1)}\sin(nx)\right)$$

b). Ekspansikan $f(x)$ ke dalam deret Cosinus Fourier untuk $0 \leq x \leq \pi$

※ Cosinus Fourier : $b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^\pi e^{-x}\,dx$$

$$= \frac{2}{\pi}\left[\left[-e^{-x}\right]_0^\pi\right]$$

$$= \frac{2}{\pi}\left[-e^{-\pi} + 1\right]$$

$$= \frac{2}{\pi}\left[\frac{e^\pi - 1}{e^\pi}\right]$$

$$a_n = \frac{2}{\pi}\int_0^\pi e^{-x} \cdot \cos(nx)\,dx$$

※ Anggap : $\int_0^\pi e^{-x}\cos(nx)\,dx = \int_0^\pi \underbrace{\cos(nx)}_{u} \cdot \underbrace{e^{-x}\,dx}_{dv} = T$

misal : $u = \cos(nx)$ $\qquad dv = e^{-x}\,dx$

$du = -n\sin(nx)\,dx$ $\qquad v = -e^{-x}$

$$T = (-e^{-x})\cos nx \Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}).(-n\sin(nx))dx$$

$$= \left[(-e^{-\pi})\cos n\pi + 1\right] - n\int_0^{\pi} (e^{-x})\sin(nx)\,dx$$

※ misal : $e^{-x}\,dx = dv$ $\quad$ $\sin(nx) = u$

$-e^{-x} = v$ $\quad$ $du = n\cos nx\,dx$

$$\int_0^{\pi} e^{-x}\sin(nx)\,dx = (-e^{-x})\sin(nx)\Big]_0^{\pi} - \int_0^{\pi} -(e^{-x}).\,n\cos nx\,dx$$

$$= 0 + n\int_0^{\pi} e^{-x}.\cos nx\,dx$$

$$= nT$$

$$\Rightarrow T = -\frac{(-1)^n}{e^{\pi}} + 1 - n(nT)$$

$$= \frac{e^{\pi} - (-1)^n}{e^{\pi}} - n^2 T$$

$$T(n^2+1) = \frac{e^{\pi} - (-1)^n}{e^{\pi}}$$

$$T = \frac{e^{\pi} - (-1)^n}{(n^2+1)e^{\pi}}$$

$$a_n = \frac{2}{\pi}.T$$

$$= \frac{2}{\pi}\left[\frac{e^{\pi} - (-1)^n}{(n^2+1)e^{\pi}}\right]$$

Jawab : $f(x) = \frac{1}{2}.\frac{2}{\pi}\left(\frac{e^{\pi}-1}{e^{\pi}}\right) + \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\left(\frac{e^{\pi} - (-1)^n}{e^{\pi}(n^2+1)}\right)\cos(nx)\right)$

$$= \frac{(e^{\pi}-1)}{\pi e^{\pi}} + \sum_{n=1}^{\infty}\left(\frac{2}{\pi}\left(\frac{e^{\pi} - (-1)^n}{e^{\pi}(n^2+1)}\right)\cos(nx)\right)$$

c). Ekspansikan $f(x)$ ke dalam deret Sinus Fourier untuk $0 \leq x < \pi$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-x}\sin(nx)\,dx$$

※ Anggap: $\int_0^{\pi} e^{-x}\sin(nx)\,dx = T$

$U = \sin(nx)$ $\quad$ $dv = e^{-x}dx$

$du = n\cos(nx)dx$ $\quad$ $v = -e^{-x}$

$$T = (-e^{-x}).\sin nx\Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}).\,n\cos(nx)dx$$

$$= \left[(-e^{-\pi}.(0)) - 0\right] + n\int_0^{\pi} e^{-x}.\cos nx\,dx$$

$$\Rightarrow \int_0^{\pi} e^{-x}.\cos nx\,dx =$$

misal : $u = \cos nx$ $\qquad dv = e^{-x}dx$

$du = -n \sin nx \, dx \qquad v = -e^{-x}$

$$\int_0^{\pi} e^{-x} \cdot \cos nx \, dx = (-e^{-x})\cos nx \Big]_0^{\pi} - \int_0^{\pi} (-e^{-x}) \cdot (-n \sin nx) \, dx$$

$$= \left(-e^{-\pi} \cdot \cos n\pi + 1\right) - n \int_0^{\pi} e^{-x} \sin nx \, dx$$

$$= -\frac{\cos n\pi}{e^{\pi}} + \frac{e^{\pi}}{e^{\pi}} - n\,T$$

$$T = n\left[\frac{e^{\pi} - \cos n\pi}{e^{\pi}} - nT\right]$$

$$T(n^2+1) = \frac{e^{\pi} - \cos n\pi}{e^{\pi}}$$

$$T = \frac{e^{\pi} - (-1)^n}{(n^2+1)\,e^{\pi}}$$

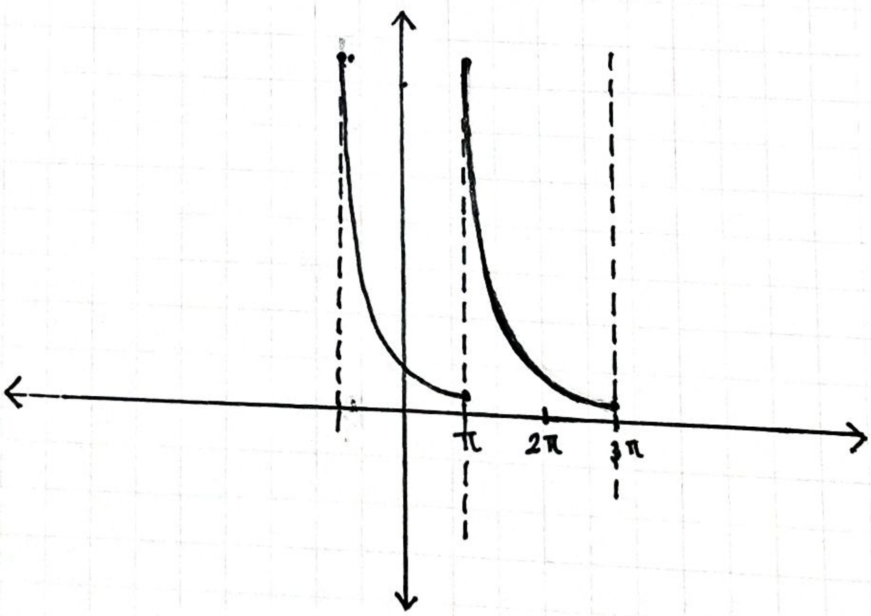
$$b_n = \frac{2}{\pi} T$$

$$= \frac{2}{\pi}\left[\frac{e^{\pi} - (-1)^n}{(n^2+1)\,e^{\pi}}\right]$$

Jawab = $f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi}\left[\frac{e^{\pi} - (-1)^n}{(n^2+1)\,e^{\pi}}\right]$ //

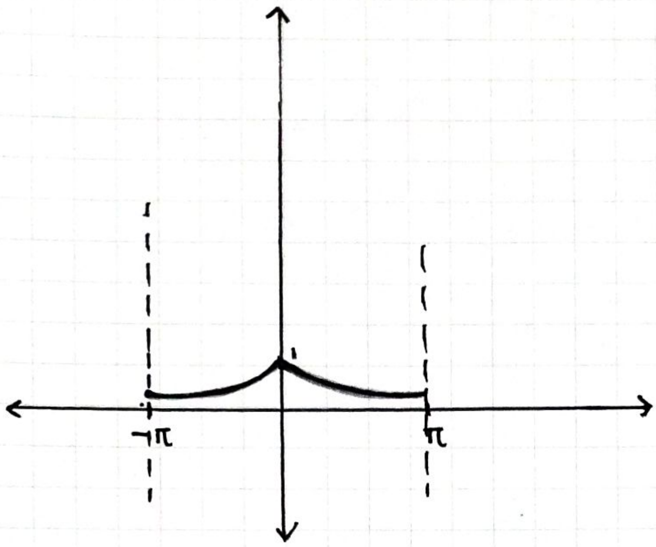
d). Gambar dari fungsi a), b), dan c) untuk 2 gelombang.

- Gambar a.



V

b). Gambar b.



c). Gambar C